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Open system approach to neutrino oscillations

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Abstract: Neutrino oscillations are studied in the general framework of open quantum systems by means of extended dynamics that take into account possible dissipative effects. These new phenomena induce modifications in the neutrino oscillation pattern that in general can be parametrized by means of six phenomenological constants. Although very small, stringent bounds on these parameters are likely to be given by future planned neutrino experiments.

Keywords: Solar and Atmospheric Neutrinos, Neutrino Physics.

Contents

1. Introduction

A large variety of open quantum systems can be modeled as being subsystems in interaction with large environments. The time evolution of the total system, subsystem plus environment, is unitary and follows the standard rules of quantum mechanics. However, the dynamics of the subsystem alone, obtained by eliminating the environment degrees of freedom, is no longer unitary, as it develops dissipation and irreversibility [1]–[3].

When there are no initial correlations between subsystem and environment and their mutual interactions can be considered weak, the resulting subdynamics can be described in terms of so-called quantum dynamical semigroups. These are time-evolution maps that encode very general physical requirements, like entropy increase, forward in time composition law (semigroup property) and complete positivity; these properties are essential for the correct physical interpretation of the subdynamics.

Although this description of open quantum systems has been originally developed in the framework of quantum optics [4]–[6], it is very general and can be applied to model a variety of different phenomena. Recently, it has been adopted to study the effects of dissipation and irreversibility in various particle physics phenomena [7]–[14].

The original motivations for such investigations were based on quantum gravity effects [15]–[20]: due to the quantum fluctuations of the gravitational field and the appearance of virtual black holes, spacetime becomes "foamy" at Planck's scale, leading to possible loss of quantum coherence. From a more fundamental point of view, also string theory could lead to similar effects in the low energy domain [21]. Indeed, subdynamics described by quantum dynamical semigroups are the result of the interaction with a "gas" of D0-branes at Planck's temperature, obeying infinite statistics [7]. Nevertheless, since not enough details about the "microscopic" dynamics are known to allow precise estimates of the magnitude of these new effects, the description of dissipative phenomena that we shall discuss below should be thought as being phenomenological in nature.

These new, non-standard effects are very small, since they are suppressed by inverse powers of the Planck mass, as a rough dimensional estimate suggests, and therefore very difficult to observe in practice. However, there are particular situations, involving interference phenomena, in which they might be in the reach of present or future experiments. Indeed, detailed studies of neutral meson systems [8, 10, 13], and neutron interferometry [14], using quantum dynamical semigroups have already been performed and order of magnitude limits on some of these dissipative effects have been derived using available experimental data [9, 11, 14]. One of the most interesting outcome of these investigations is that future experiments, in particular those involving correlated neutral mesons, should be able to ascertain with high accuracy the presence of such dissipative phenomena.

Neutrino physics is certainly another obvious place where to look for non-standard effects. Many neutrino experiments are presently taking data and other will start operating in the near future, so that it appears timely to discuss in detail to what extent dissipation can affect those observations.

We shall limit our considerations to the vacuum oscillations of two species of neutrinos. In this case, possible dissipative effects can be parametrized in terms of six phenomenological constants that modify the pattern of the transition probability $\mathcal{P}$ among the two neutrino flavours, by introducing exponential dumping factors. Although the explicit expression of $\mathcal{P}$ is in general rather complicated, in the generic case its asymptotic (large time) behaviour turns out to be independent from the mixing angle. Various approximated expressions for $\mathcal{P}$ will also be discussed; they can be of help in fitting the experimental data. Finally, in the last section we shall present a discussion on a possible physical mechanism that could be at the origin of the dissipative phenomena.

2. Quantum dynamical semigroups and neutrino oscillations

Quite in general, states of a quantum system evolving in time can be described by a density matrix ρ; this is a hermitean, positive operator, i.e. with positive eigenvalues, and constant trace. We shall analyze the evolution of neutrinos created in a given flavour by the weak interactions and subsequently detected at a later time. Assuming the neutrinos to be ultrarelativistic, the study of the transition probability for the original tagged neutrinos to be found in a different flavour can be performed using an effective description [22]–[24]; further, for simplicity, we shall limit our considerations to the mixing of two neutrino species. Then the neutrino system can be modeled by means of a two-dimensional Hilbert space, taking as basis states the two mass eigenstates.

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With respect to this basis, the two flavour states, that conventionally we shall call "ν_e" and "ν_μ", are represented by the following 2×2 matrices:

$$\rho_{\nu_e} = \begin{pmatrix} \cos^2\theta & \cos\theta\,\sin\theta \\ \cos\theta\,\sin\theta & \sin^2\theta \end{pmatrix},$$
$$\rho_{\nu_\mu} = \begin{pmatrix} \sin^2\theta & -\cos\theta\,\sin\theta \\ -\cos\theta\,\sin\theta & \cos^2\theta \end{pmatrix} \equiv 1 - \rho_{\nu_e}\,, \tag{2.1}$$

where θ is the mixing angle.

As explained in the introductory remarks, our analysis is based on the hypothesis that the evolution in time of the neutrino state ρ is given by a quantum dynamical semigroup, i.e. by a completely positive, trace-preserving family of linear maps: $\rho(0) \mapsto \rho(t)$. These maps are generated by equations of the following form:

$$\frac{\partial\rho(t)}{\partial t} = -iH_{\rm eff}\,\rho(t) + i\rho(t)\,H_{\rm eff} + L[\rho(t)]\,. \tag{2.2}$$

The first two terms in the r.h.s. of this equation are the standard quantum mechanical ones, that give rise to the traditional description of neutrino oscillations. They contain the effective (time-independent) hamiltonian $H_{\rm eff}$; neglecting effects due to possible neutrino instability, it can be taken to be hermitean. The additional piece $L[\rho]$ is a linear map, whose form is completely fixed by the conditions of complete positivity and trace conservation [1]:

$$L[\rho] = -\frac{1}{2}\sum_j \left(A_j^\dagger A_j\,\rho + \rho\,A_j^\dagger A_j\right) + \sum_j A_j\,\rho\,A_j^\dagger\,, \tag{2.3}$$

where the operators A_j must be such that $\sum_j A_j^\dagger A_j$ is a well-defined 2×2 matrix. The additional requirement of entropy increase can be easily implemented by taking the A_j to be hermitean [8]. It should be stressed that in absence of $L[\rho]$, pure states (i.e. states of the form $|\psi\rangle\langle\psi|$) would be transformed into pure states. Only when the extra piece $L[\rho]$ is also present, $\rho(t)$ becomes less ordered in time due to a mixing-enhancing mechanism: it produces dissipation and irreversibility, and possible loss of quantum coherence.

As already mentioned, equations of the form (2.2) and (2.3) have been used to describe various phenomena related to open quantum systems; in particular, they have been applied to analyze the propagation and decay of neutral meson systems [8]–[13]. Although the basic general idea behind these treatments is that quantum phenomena at Planck's scale produce loss of phase-coherence, one should keep in mind that the form (2.2), (2.3) of the evolution equations is independent from the microscopic mechanism responsible for the dissipative effects. Indeed, it is the result of very basic physical requirements that the complete time evolution, $\gamma_t : \rho(0) \mapsto \rho(t)$, needs to satisfy; generally, the one parameter (=time) family of linear maps γ_t should transform density matrices into density matrices and have the properties of increasing the

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von Neumann entropy, $S = -\,\mathrm{Tr}[\rho \ln \rho]$, of obeying the semigroup composition law, $\gamma_t[\rho(t')] = \rho(t+t')$, for t, $t' \geq 0$, of being completely positive [1]–[3]. In view of this, the equation (2.2), (2.3) can be regarded as phenomenological in nature; nevertheless, possible physical mechanisms leading these equations will be discussed in the final section.

Among the just mentioned physical requirements, complete positivity is perhaps the less intuitive. Indeed, it has not been enforced in previous analysis, in favor of the more obvious simple positivity. Simple positivity is in fact generally enough to guarantee that the eigenvalues of the density matrix $\rho(t)$ remain positive at any time; this requirement is obviously crucial for the consistency of the formalism, in view of the interpretation of the eigenvalues of $\rho(t)$ as probabilities.

Complete positivity is a stronger property, in the sense that it assures the positivity of the density matrix describing the states of a larger system, obtained by coupling in a trivial way the neutrino system with another arbitrary finite-dimensional one. Although trivially satisfied by standard quantum mechanical (unitary) time-evolutions, the requirement of complete positivity seems at first a mere technical complication. Nevertheless, it turns out to be essential in properly treating correlated systems, like two spin-zero neutral mesons coming from the decay of a vector-meson resonance; it assures the absence of unphysical effects, like the appearance of negative probabilities, that could occur for just simply positive dynamics [25]. For these reasons, in analyzing possible non-standard, dissipative effects even in simpler, non correlated systems, the phenomenological equations (2.2) and (2.3) should always be used.[1]

In the case of the neutrino system, a more explicit description of (2.2) and (2.3) can be given. In the chosen basis, the effective hamiltonian that gives rise to the standard vacuum oscillations can be written as [22]–[24]:

$$H_{\mathrm{eff}} = \begin{pmatrix} E+\omega & 0 \\ 0 & E-\omega \end{pmatrix}, \tag{2.4}$$

where E is the average neutrino energy, while $\omega = \Delta m^2/4E$ encodes the level splitting due to the square mass difference Δm^2 of the two mass eigenstates. In the case of oscillations in matter, H_{eff} has a more complicated expression, that takes into account the coherent interactions of the neutrinos with the matter constituents. For simplicity, in the following we shall limit our discussion to vacuum oscillations: we are in fact interested in studying possible dissipative effects, which are quite independent from the specific form of the standard effective hamiltonian.

[1]We have argued before (see also the discussion in section 5) that the microscopic mechanism leading to the non-standard, dissipative phenomena are likely to originate from quantum gravity or string effects. They presumably act in the same way for all systems; it is therefore unjustified to adopt different formulations for correlated and uncorrelated systems.

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The explicit expression of the term $L[\rho]$ in (2.3) can be most simply given by expanding the 2×2 matrix ρ in terms of Pauli matrices σ_i and the identity σ_0: $\rho = \rho_\mu \sigma_\mu$, $\mu = 0, 1, 2, 3$. In this way, the map $L[\rho]$ can be represented by a symmetric 4×4 matrix $[L_{\mu\nu}]$, acting on the column vector with components $(\rho_0, \rho_1, \rho_2, \rho_3)$. It can be parametrized by the six real constants a, b, c, α, β, and γ [8]:

$$[L_{\mu\nu}] = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix}, \tag{2.5}$$

with a, α and γ non-negative. These parameters are not all independent; the condition of complete positivity of the time-evolution $\rho \to \rho(t)$ imposes the following inequalities:

$$\begin{aligned} &2\,R \equiv \alpha + \gamma - a \geq 0 \qquad && U \equiv RS - b^2 \geq 0 \\ &2\,S \equiv a + \gamma - \alpha \geq 0 && V \equiv RT - c^2 \geq 0 \\ &2\,T \equiv a + \alpha - \gamma \geq 0 && Z \equiv ST - \beta^2 \geq 0 \\ &X \equiv RST - 2\,bc\beta - R\beta^2 - Sc^2 - Tb^2 \geq 0\,. \end{aligned} \tag{2.6}$$

Taking into account that the equation in (2.2) is trace preserving, from the initial normalization condition $\mathrm{Tr}[\rho(0)] = 1$, one immediately obtains that $\rho_0 = 1/2$, for all times. Then, the evolution equation for the remaining three components of $\rho(t)$ can be compactly rewritten in a Schrödinger-like form:

$$\frac{\partial}{\partial t} |\rho(t)\rangle = -2\,\mathcal{H}\,|\rho(t)\rangle\,, \tag{2.7}$$

where the vector $|\rho(t)\rangle$ has components (ρ_1, ρ_2, ρ_3), and

$$\mathcal{H} = \begin{pmatrix} a & b+\omega & c \\ b-\omega & \alpha & \beta \\ c & \beta & \gamma \end{pmatrix}. \tag{2.8}$$

The formal solution of (2.7) involves the exponentiation of the matrix $\mathcal{H}$:

$$|\rho(t)\rangle = M(t)\,|\rho(0)\rangle\,, \qquad M(t) = e^{-2\,\mathcal{H}\,t}\,. \tag{2.9}$$

Let us assume that at the beginning the neutrinos were of type "ν_e"; the probability of having a transition into the type "ν_μ" at time t is given in our formalism by:

$$\mathcal{P}_{\nu_e \to \nu_\mu}(t) = \mathrm{Tr}[\rho_{\nu_e}(t)\,\rho_{\nu_\mu}]\,, \tag{2.10}$$

where $\rho_{\nu_e}(t)$ is the solution of (2.7) with the initial condition given by the matrix ρ_{ν_e} in (2.1). Using (2.9) and the matrices in (2.1), one explicitly finds:

$$\begin{aligned} \mathcal{P}_{\nu_e \to \nu_\mu}(t) = \frac{1}{2}\Big\{ &\cos^2 2\theta\,[1 - M_{33}(t)] + \sin^2 2\theta\,[1 - M_{11}(t)] - \\ &- \frac{1}{2} \sin 4\theta\,[M_{13}(t) + M_{31}(t)] \Big\}\,. \end{aligned} \tag{2.11}$$

When the additional piece $L[\rho]$ in (2.3) is not present, one simply obtains:

$$M_{11}(t) = \cos(2\,\omega t)\,, \qquad M_{13}(t) + M_{31}(t) = 0\,, \qquad M_{33}(t) = 1\,, \tag{2.12}$$

so that (2.11) reduces to the well known standard expression for the oscillation probability in vacuum:

$$\mathcal{P}^{(0)}_{\nu_e\to\nu_\mu}(t) = \sin^2 2\theta\, \sin^2 \omega t\,. \tag{2.13}$$

Therefore, any deviation from (2.12) that might be found in fitting the expression (2.11) with data from neutrino experiments would provide evidence for dissipative phenomena in neutrino physics.[2]

3. Transition probability: general properties

Explicit expressions for the entries of the matrix $M(t)$ appearing in (2.9) can be given by diagonalizing $\mathcal{H}$ in (2.8); this can always be done by solving the corresponding eigenvalue equation,

$$\mathcal{H}\,|v^{(k)}\rangle = \lambda^{(k)}\,|v^{(k)}\rangle\,, \qquad k = 1,2,3\,, \tag{3.1}$$

via Cardano's formula.[26] Then, using the diagonalizing matrix $[D_{\ell k}] \equiv {v^{(k)}}_\ell$, built with the components of the eigenvectors $|v^{(k)}\rangle$, one formally writes:

$$M_{ij}(t) = \sum_{k=1}^{3} e^{-2\lambda^{(k)}t}\; D_{ik}\, D^{-1}_{kj}\,. \tag{3.2}$$

The explicit expressions of $\lambda^{(k)}$ and $[D_{\ell k}]$ in terms of the dissipative parameters a, b, c, α, β, γ and ω is however cumbersome, making the formula (3.2) unmanageable in practice; for this reason, we shall discuss particularly interesting limits of the general expression (2.11) for the transition probability $\mathcal{P}_{\nu_e\to\nu_\mu}(t)$ in the next section. Nevertheless, general conclusions on the behaviour of (3.2) can be obtained by studying in more detail the eigenvalue problem in (3.1).

The three eigenvalues $\lambda^{(1)}$, $\lambda^{(2)}$, $\lambda^{(3)}$ of the matrix $\mathcal{H}$ are solutions of the cubic equation:

$$\lambda^3 + r\,\lambda^2 + s\,\lambda + w = 0\,, \tag{3.3}$$

with real coefficients,

$$\begin{aligned} r &\equiv -(\lambda^{(1)} + \lambda^{(2)} + \lambda^{(3)}) = -(a + \alpha + \gamma) \\ s &\equiv \lambda^{(1)}\lambda^{(2)} + \lambda^{(1)}\lambda^{(3)} + \lambda^{(2)}\lambda^{(3)} = a\alpha + a\gamma + \alpha\gamma - b^2 - c^2 - \beta^2 + \omega^2 \\ w &\equiv -\lambda^{(1)}\lambda^{(2)}\lambda^{(3)} = a\beta^2 + \alpha c^2 + \gamma(b^2 - \omega^2) - a\alpha\gamma - 2\,bc\beta\,. \end{aligned} \tag{3.4}$$

[2]Different physical mechanisms have been proposed in the literature to account for the observed neutrino flux deficit: they all predict expressions for the transition probability $\mathcal{P}_{\nu_e\to\nu_\mu}(t)$ that differ from that in (2.11); see the discussion at the end of section 5.

According to the sign of the associated discriminant $\mathcal{D} = p^3 + q^2$, $p = s/3 - (r/3)^2$, $q = (r/3)^3 - rs/6 + w/2$, the eigenvalues are either all real ($\mathcal{D} \leq 0$), or one is real and the remaining two are complex conjugate ($\mathcal{D} > 0$). The degenerate case $\mathcal{D} = 0$ occurs when two real eigenvalues are equal; all three coincide for $p = q = 0$.

Furthermore, the quantum dynamical semigroup generated by (2.2), (2.3) is bounded for any t [27], so that the real parts of $\lambda^{(1)}$, $\lambda^{(2)}$, $\lambda^{(3)}$ are surely non-negative (otherwise the entries $M_{ij}(t)$ in (3.2) would blow up for large times).

When $\omega = 0$, the matrix $\mathcal{H}$ is real, symmetric and non-negative, as guaranteed by the inequalities (2.6); therefore, its eigenvalues are all real and non-negative: $\mathcal{D} < 0$ and this is possible only for $p < 0$. The discriminant $\mathcal{D}$ starts becoming positive only for sufficiently large ω, since, as it is clear from the definitions (3.4), the contribution of ω to p is equal to $\omega^2/3$, and thus it is positive.

Therefore, the time-behaviour of the transition probability $\mathcal{P}_{\nu_e\to\nu_\mu}(t)$ depends on the relative magnitude of ω with respect to the non-standard parameters a, b, c, α, β and γ. In particular, an oscillatory behaviour is possible only when the dissipative parameters are small compared to ω; on the other hand, when dissipation is the dominant phenomenon, the time-dependence in (3.2), and therefore in (2.11), is characterized by exponential dumping terms.

This analysis allows a general discussion on the asymptotic behaviour of $\mathcal{P}_{\nu_e\to\nu_\mu}(t)$ for large t [27, 1]. In the generic case, $\det(\mathcal{H}) \neq 0$ and all three eigenvalues $\lambda^{(1)}$, $\lambda^{(2)}$, $\lambda^{(3)}$ are thus non-vanishing, with positive (or zero) real part, as discussed above. When $\mathcal{D} \leq 0$, the eigenvalues are all real, so that all entries of the matrix $M(t)$ in (3.2) approach zero for large t, due to the exponential dumping factors. The same is true also in presence of two complex conjugate eigenvalues, unless their real part is identically zero. However, this situation never occurs when there is a non-vanishing dissipative contribution (2.5) in the equation (2.2). Indeed, from (3.4) one finds that the condition for having two purely imaginary eigenvalues is given by: $w - rs = 0$; recalling the definitions in (2.6), it can be rewritten as: $X + (R+S+T)[U+V+Z] + 2\,(R+S+T)^3 + \omega^2(R+S+2T) = 0$. Since by the inequalities in (2.6) all the terms in the l.h.s. are non negative, they must be zero separately, which is possible only for $a = b = c = \alpha = \beta = \gamma = 0$.

Therefore, in presence of dissipative phenomena, the generic large t behaviour of the transition probability in (2.11) is independent from the mixing angle θ:

$$\mathcal{P}_{\nu_e\to\nu_\mu}(t) \underset{t\to\infty}{\sim} \frac{1}{2}\,. \tag{3.5}$$

The situation might be different however when $\det(\mathcal{H}) = 0$ and we are in presence of zero eigenvalues. In this special case, ω and the dissipative parameters a, b, c, α, β and γ need to satisfy the additional cubic condition $w = 0$. Keeping ω arbitrary, the only way to satisfy this constraint is to set $\gamma = 0$; indeed, the inequalities (2.6) immediately imply: $b = c = \beta = 0$ and $a = \alpha$ and therefore a vanishing w.

The matrix $\mathcal{H}$ in (2.8) takes now a very simple form, and the non-vanishing eigenvalues are complex: $\lambda^{(1)}, \lambda^{(2)} = \alpha \pm i\omega$. Since α is positive, most of the entries of the evolution matrix $M(t)$ in (3.2) are still exponentially suppressed for large t; however, the presence of the zero eigenvalue now implies $M_{33}(t) = 1$, so that the asymptotic form of (2.11) changes:

$$\mathcal{P}_{\nu_e \to \nu_\mu}(t) \underset{t\to\infty}{\sim} \frac{1}{2} \sin^2 2\theta \,. \tag{3.6}$$

The large-time behaviors (3.5) and (3.6), for the particular case $\gamma = 0$, are characteristic of the presence of the dissipative contribution (2.5) to the evolution equation (2.2). However, in general, it might be very difficult to distinguish these behaviours from the one obtained in the standard case. Although in principle $\mathcal{P}^{(0)}_{\nu_e \to \nu_\mu}(t)$ in (2.13) has a purely oscillatory form, in any actual observational condition, the oscillations are likely to be averaged away, so that also in this case (3.6) holds. Therefore, when the mixing is maximal ($\sin^2 2\theta \approx 1$), or in the special situation in which only one dissipative parameter is non-vanishing ($\gamma = 0$), the asymptotic large t behaviors (3.5) and (3.6) turn out to be indistinguishable from that of $\mathcal{P}^{(0)}_{\nu_e \to \nu_\mu}(t)$. In these cases, one has to study the full time dependence of the transition probability.

4. Transition probability: explicit form

The general expression of the transition probability $\mathcal{P}_{\nu_e \to \nu_\mu}(t)$ in terms of ω and the dissipative parameters is very complicated and not particularly useful in practical applications. Therefore, we shall now discuss some approximations for which $\mathcal{P}_{\nu_e \to \nu_\mu}(t)$ assumes a more manageable form; it might be of interest to compare these expressions with actual experimental data in order to put limits on the magnitude of the dissipative constants. Although this is clearly beyond the scope of the present investigation, we shall nevertheless briefly comment about the rough sensitivity that one might expect from the analysis of present and future experiments.

As discussed before, in general $\mathcal{P}_{\nu_e \to \nu_\mu}(t)$ contains two kind of contributions: oscillating terms, controlled by ω, and exponentially dumping terms, signaling dissipative effects. The relative dominance of these two types of behaviour depends on the magnitude of a, b, c, α, β and γ when compared to ω.

In our approach, the dissipative contribution (2.5) to the evolution equation (2.2) should be regarded as phenomenological; it is therefore hard to give an apriori estimate of the magnitude of the non-standard parameters in $L[\rho]$. As mentioned in the Introduction and further discussed in the next section, a general framework in which dissipative effects naturally emerge is provided by the study of open quantum systems, i.e. systems in weak interactions with large environments. In such cases the dissipative effects can be roughly estimated to be proportional to the typical energy scale of the system, while suppressed by inverse powers of the characteristic energy scale of the environment [1, 2, 3, 16, 7].

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In the case of the neutrino system, on the basis of these considerations and in line with the idea that dissipation is induced by quantum effects at Planck's scale, one expects the values of the parameters a, b, c, α, β and γ in (2.5) to be very small; for any fixed neutrino source and observational conditions, an upper bound on the magnitude of these parameters can be roughly evaluated to be of order E^2/M_P, with M_P the Planck mass. The ratio of a, b, c, α, β and γ with ω can thus be estimated to be at most of order $10^{-10}\ E^3/\Delta m^2$, with E expressed in MeV and the neutrino square mass difference Δm^2 in eV^2. By taking for E and Δm^2 values that are typical of various neutrino sources, this ratio turns out to be about 10^2 for atmospheric neutrinos, of order one for solar neutrinos, while for accelerator neutrinos it can be as small as 10^{-2}.

When the dissipative, non-standard parameters are large or of the same order of magnitude of ω, all entries of $\mathcal{H}$ in (2.8) are in general different from zero. In this case a useful approximation is to assume c and β to be much smaller than the remaining constants.[3] To lowest order, the matrix $\mathcal{H}$ becomes block diagonal and a manageable expression for the transition probability in (2.11) can be obtained. Explicitly, one finds:

$$\mathcal{P}_{\nu_e\to\nu_\mu}(t) = \frac{1}{2}\Bigg\{\cos^2 2\theta[1-e^{-2\gamma t}] + \\ + \sin^2 2\theta\left[1 - e^{-At}\left(\cos(2\Omega_0 t) + \frac{\mathrm{Re}\,B}{2\Omega_0}\sin(2\Omega_0 t)\right)\right]\Bigg\}, \qquad (4.1)$$

where

$$A = \alpha + a\,, \qquad B = \alpha - a + 2ib\,, \qquad \Omega_0 = \sqrt{\omega^2 - \frac{|B|^2}{4}}\,. \qquad (4.2)$$

The oscillating behavior in (4.1) depends on the magnitude of ω with respect to $|B|$; when $\omega < |B|/4$, the frequency Ω_0 becomes purely imaginary and $\mathcal{P}_{\nu_e\to\nu_\mu}(t)$ contains only exponential terms. In any case, the exponential dumping terms in (4.1) dominate for large t, and the limit (3.5) is recovered.

A further simplification occurs when $\gamma = 0$; as already observed in the previous section, this automatically guarantees $c = \beta = 0$, an further imposes $b = 0$ and $a = \alpha$. In this case, (4.1) reduces to:

$$\mathcal{P}_{\nu_e\to\nu_\mu}(t) = \frac{1}{2}\sin^2 2\theta\left[1 - e^{-2\alpha t}\cos(2\,\omega t)\right]. \qquad (4.3)$$

This is the most simple form that the transition probability formula takes in presence of dissipative effects: with respect to the standard expression in (2.13) and (4.3) contains an exponential dumping factor in front of the oscillating term. It can be used to derive the rough order of magnitude bound on the non-standard parameter

[3]Note that this choice is perfectly compatible with the constraints of complete positivity given in (2.6)

α that can be expected from neutrino experiments. Assuming that the dumping due to the exponential term is not exceeding a few percent, from (4.3) one derives: $\alpha t \leq 1$. Since the neutrinos are relativistic, the flight-time between emission and detection is roughly the same as the distance ℓ between source and detector. Then, one has: $\alpha \leq 1/\ell$, where $1/\ell$ is approximately 10^{-22} GeV, 10^{-24} GeV, 10^{-27} GeV for accelerator, atmospheric, solar neutrinos, respectively. Although the best bound on α seems to be given by solar neutrinos experiments, due to the larger ℓ, atmospheric neutrinos data are the most suitable for a meaningful fit of (4.3), since in this case its time (or ℓ) dependence can actually be probed.

Another very useful approximation of the general formula (2.11) for the transition probability can be obtained when the non-standard parameters a, b, c, α, β and γ are small compared with ω. In this case, the additional dissipative term $L[\rho]$ in (2.2) can be treated as a perturbation. Then, up to second order in the small parameters, one explicitly gets:

$$\begin{aligned}\mathcal{P}_{\nu_e\to\nu_\mu}(t) = \frac{1}{2}\Bigg\{ &\cos^2 2\theta\left[1 - e^{-2\gamma t}\left(1 + \frac{2|C|^2}{\Omega^2}\sin^2(\Omega t)\right)\right] + \\ &+ \sin^2 2\theta\left[1 - e^{-At}\left(\cos(2\Omega t) + \frac{\mathrm{Re}\,B}{2\Omega}\sin(2\Omega t) - \frac{2(\mathrm{Im}\,C)^2}{\Omega^2}\sin^2(\Omega t)\right)\right] + \\ &+ \sin 4\theta\, e^{-At}\left[\frac{\mathrm{Re}\,C}{\Omega}\sin(2\Omega t) + \frac{\mathrm{Re}[C(A+B-2\gamma)]}{\Omega^2}\sin^2(\Omega t)\right]\Bigg\}\,, \end{aligned} \tag{4.4}$$

where A and B are as in (4.2), while:

$$C = c + i\beta\,, \qquad \Omega = \sqrt{\omega^2 - |C|^2 - \frac{|B|^2}{4}}\,. \tag{4.5}$$

In the previous formula, we have reconstructed the exponential factors by consistently putting together the terms linear and quadratic in t; a similar treatment has allowed writing the oscillatory contributions in terms of the frequency Ω.[4]

As a further check, note that the expression (4.4) reduces to that in (4.2) for $|C| = 0$, i.e. when $c = \beta = 0$: it is therefore a correction to (4.2) for nonvanishing C. In this respect, the validity of (4.4) goes beyond the approximation in which it has been derived, since it can be considered as the expansion up to second order of the general formula (2.11) for c and β small. Therefore, it can be used with confidence in fitting experimental data from neutrino oscillation experiments.

In this respect, the data on atmospheric neutrinos are presently the best place to look for dissipative effects. Applying techniques similar to the ones employed e.g. in [28] and [29] to the generalized transition probability (4.4), one should be able to extract from the actual data useful bounds on some of the non-standard parameters in (2.5). Nevertheless, one should note that having in general six additional unknowns to fit will certainly make the procedure much more difficult and complex than in

[4] This frequency is now real, since by hypothesis $\omega^2 \gg |C|^2 + |B|^2/4$.

the standard case, where only the mixing angle θ and the mass difference Δm^2 are present; only for the simplified expression (4.3), that contains just one additional parameter besides θ and Δm^2, one can actually expect a good fitting accuracy.

5. Discussion

In the previous sections we have discussed how a phenomenological approach based on quantum dynamical semigroup can be used to describe dissipative dynamics for the neutrino system. As already mentioned in the introductory remarks, this phenomenological treatment can be supported by physical considerations. Indeed, a general picture in which dissipative effects naturally emerge is provided by systems in weak interaction with suitable environments. In the case of elementary particle systems, these effects are likely to originate from the dynamics of strings; however, an effective description of the environment, encoding some of the "collective" properties of the underlying fundamental theory, is quite adequate for a more physical discussion of evolutions of type (2.2), (2.3) [7].

To be more specific, in the case of neutrino systems, the total hamiltonian can always be decomposed as:

$$H_{\rm tot} = H_{\rm eff} \otimes \mathbb{1} + \mathbb{1} \otimes H_{\mathcal{E}} + g\, H' \,, \tag{5.1}$$

where $H_{\rm eff}$ is as in (2.4), while $H_{\mathcal{E}}$ describes the internal dynamics of the environment $\mathcal{E}$. The interaction terms between the two systems are assumed to be weak: they are encoded in H', with g a small coupling constant.

Furthermore, the mechanism of neutrino production is different from the one responsible for the dissipative effects; it is therefore natural to assume that the neutrino state and that of the environment be uncorrelated at the moment of the neutrino emission. In other words, the initial state of the total system can be taken to be in factorized form: $\rho_{\rm tot} = \rho \otimes \rho_{\mathcal{E}}$.

The time evolution of the neutrino state ρ, obtained by tracing over the environment degrees of freedom,

$$\rho \mapsto \rho(t) = {\rm Tr}_{\mathcal{E}} \left[e^{-iH_{\rm tot}t} \left(\rho \otimes \rho_{\mathcal{E}}\right) e^{iH_{\rm tot}t} \right] , \tag{5.2}$$

is in general very complicated and can not be described explicitly. Nevertheless, an evolution equation of the form (2.2), (2.3) for $\rho(t)$ naturally emerges by taking into account the physical requirement that the interaction between neutrinos and environment be weak.

There are essentially two different ways of implementing this condition [1]–[3]: they correspond to the two ways of making the ratio $\tau/\tau_{\mathcal{E}}$ large, where τ is the typical variation time of $\rho(t)$, while $\tau_{\mathcal{E}}$ represents the typical decay time of the correlations in the environment. Indeed, only for $\tau \gg \tau_{\mathcal{E}}$ one expects the memory effects implicitly encoded in (5.2) to be negligible, and a local in time evolution for $\rho(t)$ to be valid.

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When $\tau_{\mathcal{E}}$ becomes small, while τ remains finite, one speaks of "singular coupling limit", since the typical time-correlations of the environment approach a δ-function. In the other case, when $\tau_{\mathcal{E}}$ remains finite, while τ becomes large, one works in the framework of the so-called "weak coupling limit"; in practice, this is obtained by rescaling the time variable, $t \to t/g^2$, and sending the coupling constant g to zero (van Hove limit).

The choice between the two limits is made on the basis of physical considerations. In the case of unstable systems for instance, the weak coupling choice is unviable, since in this case τ can be identified with the (finite) lifetime. On the contrary, for neutrino systems both limits are in principle allowed.[5] They give rise to different explicit expressions for the additional contribution $L[\rho]$ in (2.3); in the case of the singular coupling limit, one finds:

$$L[\rho] = -\int_0^\infty dt\, \mathrm{Tr}_{\mathcal{E}} \left\{ \left[e^{iH_{\mathcal{E}}t}\, H'\, e^{-iH_{\mathcal{E}}t}\, , [H', \rho \otimes \rho_{\mathcal{E}}] \right] \right\}, \tag{5.3}$$

while in the weak coupling limit, one obtains:

$$L[\rho] = -\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} ds \int_0^\infty dt\, \mathrm{Tr}_{\mathcal{E}} \left\{ e^{iH_{\mathrm{eff}}s} \left[e^{iH_0 t}\, H'\, e^{-iH_0 t}\, , [H', \rho \otimes \rho_{\mathcal{E}}] \right] e^{-iH_{\mathrm{eff}}s} \right\}, \tag{5.4}$$

where H_0 is the limit of H_{tot} when the coupling constant g vanishes.

As mentioned before, the general form of the expressions for $L[\rho]$ given above does not actually depend very much on the details of the environment dynamics; an effective description that takes into account its most fundamental characteristic properties is enough to allow an explicit evaluation of the integrals in (5.3) and (5.4). Following the idea that the dissipative effects originate from the low energy string dynamics at Planck's scale, one can effectively model the environment as a gas of D0-branes, in thermodynamic equilibrium at Planck's temperature; these quanta obey an infinite statistics [30]–[32].

Explicit computations then show that both expressions (5.3) and (5.4) assumes precisely the form given in (2.5).[6] However, while in the case (5.3) all six parameters a, b, c, α, β, γ are in general nonvanishing, in the weak coupling limit the average procedure in (5.4) implies $a = \alpha$ and $b = c = \beta = \gamma = 0$, independently from the value of ω. As a consequence, when the weak coupling limit conditions are satisfied, the dissipative piece of the extended dynamics is controlled by a single parameter and

[5] Nevertheless, it should be stressed that the condition that makes the characteristic times of the neutrino system much larger than that of the environment, implicit in the weak coupling limit, might not be attainable in all situations; on the contrary, the condition on the environment time-correlations necessary for the singular coupling limit seems more natural, in view of its possible "stringy" origin.

[6] The steps followed for the evaluation of the integrals in (5.3) and (5.4) do not much differ from the ones presented in [7]; the details are therefore omitted.

the transition probability $\mathcal{P}_{\nu_e\to\nu_\mu}(t)$ assumes the simplified form presented in (4.3); on the other hand, the more general behaviour (4.4) is surely the result of a singular coupling limit procedure.

Therefore, the indication of a non-vanishing value for more than one of the parameters a, b, c, α, β, γ in neutrino oscillation experiments would certainly select the form (5.3) for the dissipative piece $L[\rho]$; in turn, this would provide some indirect informations on the structure of the environment and thus on the effective dynamics of low energy string theory.

In closing, we would like to make a few comments on the existing literature on the subject. Studies of possible phenomena violating quantum mechanics in neutrino dynamics have recently appeared [33]–[35]. Based on ideas originally presented in [16], they discuss modifications of the standard oscillation probability formula. However, the extended dynamics used in such investigations is that of [16], which does not satisfy the condition of complete positivity; as mention before, this could lead to serious inconsistencies. We stress that to avoid these problems, one has to adopt phenomenological descriptions based on the equations (2.2) and (2.5).

Kinetic evolution equations similar to the one presented in section 2 have been used to describe other, more conventional dissipative phenomena that arise due to the scattering and absorption processes in the core of supernovae or in the early universe [36]. In these extreme conditions, the frequent collisions affect the free evolutions of the neutrino species, and the consequent decoherence effects modify the oscillation pattern. The physical situation is now quite different from the one discussed in the previous sections and necessarily requires a second-quantized, field-theoretical extension of the formalism. Further, the derivation of the evolution equations can not rely on the weak-coupling limit arguments discussed above; rather, it is based on the use of specific effective interaction hamiltonians. Nevertheless, also in these cases physical requirements like the condition of complete positivity should in general be enforced and might turn out to be crucial for the self-consistency of the formalism.

Dynamical equations of the form (2.2) have further been employed for the study of the propagation of neutrinos in a density fluctuating media, in particular, in the interior of the sun [37]. They give rise to expressions for the surviving probability of the electron neutrinos that differ from those obtained in the framework of standard matter oscillations. Although described in terms of quantum dynamical semigroups, these density fluctuation have their origin in the dynamics of the sun and operate at energy scales quite different from Planck's mass. Therefore, they can be easily isolated from the dissipative effects discussed in the previous sections, that, in view of their "microscopic" origin, are not expected to be influenced by long-range phenomena.

The recent experimental data, in particular on solar and atmospheric neutrinos, show evidence of attenuation in the expected neutrino flux, signaling disappearance phenomena. Although one is led to interpret these results in terms of the standard

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oscillation formula (2.13), several other physical mechanisms have been proposed as alternative explanation for the effect, in particular: neutrino decay, flavour changing neutral currents, violation of Lorentz invariance or of the equivalence principle. In all these cases, the transition probability $\mathcal{P}$ has a dependence on time (or pathlength) and neutrino energy that differ from the standard one. (For recent discussions, see [28, 29].)

The dissipative effects studied here are clearly distinct and independent from all these explanations for the neutrino flux deficit. In particular, the dependence of $\mathcal{P}$ on the non-standard parameters a, b, c, α, β and γ is distinctive of dissipative phenomena and can not be mimicked by the other mechanisms. This is a great advantage in the process of fitting and comparing the experimental data, since it makes possible the identification of the dissipative contributions quite independently from all other effects.

Note added. After the submission of our manuscript, the paper in ref. [38] appeared; using the atmospheric neutrino data of the Super-Kamiokande experiment a bound on one of the dissipative parameters was obtained.

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